Under-Employment and the Trickle-Down of Unemployment[*]

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Abstract

A substantial fraction of workers are under-employed, i.e., employed in jobs for which they are over-qualified, and that fraction—the under-employment rate—is higher in recessions. To explain these facts, we build a search model with an endogenous "ranking" mechanism, in which high-skill applicants are systematically hired over less-skilled competing applicants. Some high-skill workers become under-employed in order to escape the competition for high-skill jobs and find a job more rapidly at the expense of less-skilled workers. Quantitatively, the model can capture the key characteristics of under-employment, notably the facts that both the under-employment rate and the wage loss associated with becoming under-employed increase in recessions.

JEL: E24, J22, J64.

The recession left millions of college-educated working in coffee shops and retail stores.[1]

While the unemployment rate is the traditional gauge of the labor market, this paper argues that it misses an important dimension of the state of the labor market: under-employment. In the US, a substantial fraction of workers are under-employed, i.e., employed in jobs for which they are over-qualified, and that fraction—the under-employment rate—is strongly counter-cyclical, increasing markedly in slack labor markets. As shown in Figure 1, the fraction of US college graduates working in lower skill-requirement occupations increased from 38.5 percent in 2008 to 41.5 percent in 2012. In other words, in those four years, the number of under-employed workers increased by 3 million; almost half as much as the increase in unemployed workers (about 7 million workers) over the same period.

While under-employment finds a large echo in the media, there is surprisingly little work on the determinants of under-employment and its implications for business cycle fluctuations. In this paper, we study the characteristics and the determinants of under-employment. First, we document new stylized facts about under-employment, notably its counter-cyclicality. Second, we provide a quantitative model that can explain the existence and counter-cyclicality of under-employment, and we use the model to explore the efficiency and distributional consequences of under-employment.

We show three stylized facts using CPS micro data: (i) under-employment is counter-cyclical, (ii) under-employment is costly, an under-employed worker earns substantially less than his non-under-employed counterpart, and (iii) under-employment is a persistent state with more than 70 percent of newly under-employed workers still under-employed one year later.

We develop a search model of under-employment in which some high-skill workers become under-employed in order to escape competition from their high-skill peers and find a job more easily. The key ingredients of our model are heterogeneity across workers and jobs, coordination frictions, and wage competition between workers. Workers differ in their skill level, islands differ in their productivity level, and workers direct their search to one island. In each island, there are coordination frictions: some vacancies will receive multiple applications while other vacancies will have no applicants, and not every worker will get a job. Importantly, when a vacancy receives multiple applications, hiring is not random: applicants compete for the job during wage bargaining, and the firm ends up hiring the most profitable applicant. The

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[1]The Wall Street Journal, 26 March 2013.

negotiated wage then depends on both the number and the skills of other applicants. Through the wage bargaining process, the model generates endogenously a "ranking" mechanism favoring high-skill job seekers, as high-skill applicants are systematically hired over less-skilled competing applicants.

We first present a simpler static version of our model that allows us to convey the main intuition and characterize analytically a number of properties of under-employment: (i) existence, (ii) counter-cyclicality, (iii) re-distributional consequences, and (iv) (in)efficiency. First, under-employment exists, not because low-qualification jobs are more abundant, but because the competition to get a low-qualification job is, from the viewpoint of high-skill workers, less intense. Second, following an adverse aggregate labor demand shock affecting all types of jobs, high-skill workers are less affected by the drop in low-requirement jobs because of their ranking advantage. As a result, they smooth the adverse aggregate shock by moving down the job ladder in greater proportion, and under-employment increases in recessions. However, this smoothing takes place at the expense of low-skill workers, and the counter-cyclicality of under-employment exacerbates the income volatility faced by the low-skilled. In other words, the counter-cyclicality of under-employment has distributional consequences, and during recessions unemployment trickles down from the high-skilled to the low-skilled. Finally, the level of under-employment is generally inefficient, and there is too much under-employment in the decentralized allocation. Although the constrained optimal allocation in which the planner allocates job seekers calls for some level of under-employment in order to maximize the matching probability of high-skill workers, the low-requirement jobs are too attractive to high-skill workers.

We then simulate a dynamic and stochastic version of our general equilibrium model with aggregate productivity shocks, and we assess its ability to quantitatively account for the stylized facts documented in the first part of the paper. The model can match key features of the US labor market; notably the level and counter-cyclicality of under-employment as well as the level and cyclicality of the wages and job finding rates of the different skill groups. In particular, consistent with the data, the model implies that both the wage cost of under-employment *and* under-employment increase during recessions.

The phenomenon of under-employment relates closely to the phenomenon of over-education,[2] which goes back to the 1970s when the supply of educated workers outpaced its demand in the labor market, apparently resulting in a substantial

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[2]Since we take the education level as given and study the resulting worker allocation problem, our focus is different from the over-education literature which mainly studies the returns from schooling.

reduction in the returns to schooling (Freeman, 1976; McGuinness, 2006). While the literature on over-education has focused on low-frequency patterns, our interest is instead on business cycle fluctuations. Our finding of a counter-cyclical under-employment rate relates to an old literature on "cyclical upgrading" —the possibility that the quality of matches improves in tighter labor markets (Reder, 1955; Okun, 1973)— and is consistent with recent evidence by Beaudry, Green and Sand (2013) and Abel and Deitz (2015) for the US.[3] More generally, the counter-cyclicality of under-employment echoes the counter-cyclicality of mismatch (Şahin et al., 2014), and our paper relates to recent work on worker mobility across occupations or industries over the business cycle (Alvarez and Shimer, 2011; Carrillo-Tudela and Visschers, 2013; Chang, 2012; Pilossoph, 2012), although we focus on vertical mobility —between high-degree requirement and low-degree requirement occupations—.

Our modeling of under-employment revives an older idea from Thurow (1975), in which individuals compete against one another for job opportunities, and in which higher educated workers can crowd out lower educated workers.[4] Relative to more recent work, our modeling of under-employment builds on the random matching literature with multiple islands and heterogeneous agents,[5] and on the competitive search literature with heterogeneous agents,[6] in which firms post wage offers and workers can direct their search to their most preferred markets. In contrast to the random matching literature, we allow firms to bargain with multiple applicants. This feature is central to generating counter-cyclical under-employment. In contrast to the competitive search literature, we relax the wage-posting assumption that firms commit to a wage and cannot negotiate a lower wage when they receive multiple applications.

To capture wage negotiations with multiple and heterogeneous applicants in a non-random hiring setting, we propose a tractable bargaining setup that embeds the bargaining outcome of two important benchmarks in the literature. When negotiation involves only one applicant, the surplus is shared as in a Nash bargaining game. When bargaining takes place with multiple applicants, our setup embeds the job auction outcome of Shimer (1999) and Julien, Kennes and King (2000) as

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[3]Note however that Gautier et al. (2002) find no evidence of cyclical upgrading in the Netherlands.

[4]Another line of research aims to explain the existence of over-education with career mobility and on-the-job search (Sicherman, 1991; Sicherman and Galor, 1990; Dolado, Jansen and Jimeno, 2009), as high-educated workers can choose to become under-employed in order to get better jobs later. However, the low exit rate out of under-employment that we document suggests that career mobility cannot be the only mechanism at play.

[5]See Albrecht and Vroman (2002); Gautier (2002); Blázquez and Jansen (2008); Dolado, Jansen and Jimeno (2009); Charlot and Decreuse (2010).

[6]See Shi (2001, 2002); Shimer (2005a); Eeckhout and Kircher (2010).

a special case.[7] The difference with Shimer (1999) and Julien, Kennes and King (2000) is how the surplus generated by the first-best applicant over the second-best applicant—the marginal surplus of the first-best applicant—is shared between the first-best applicant and the firm. In Shimer (1999) and Julien, Kennes and King (2000), the first-best applicant captures all the marginal surplus, but, in our framework, this is not necessarily the case as the marginal surplus is split between the firm and the worker.

The remainder of this paper is structured as follows. In Section 1, we study the properties of under-employment. Section 2 presents a static model of under-employment, in partial equilibrium and in general equilibrium, and discusses the optimality of the decentralized allocation. Section 3 presents a dynamic and stochastic version of our model and assesses its quantitative performances, and the final section concludes.

1 The anatomy of under-employment

This section presents three stylized facts about under-employment in the US: (i) under-employment is counter-cyclical, (ii) under-employment is a persistent state for workers who decide to move down the job ladder, (iii) moving down the job ladder implies a substantial wage loss and that wage loss is larger in recessions.

1.1 The cyclicality of under-employment

We define as under-employed an individual with some college (or more) who is employed in an occupation that requires at most a high-school degree.[8]

Occupations are defined at the 3-digit Standard Occupational Classification level. To measure a worker's education level and occupation, we use micro-data from the CPS between 1979 and 2013. To measure the degree requirement of an occupation, we use data from the BLS 2012 Occupational Outlook Handbook on the education requirements by occupation. The BLS determines the education requirement of each occupation from federal and state regulations and from the typical path of entry into a job. We keep the education requirements by occupation fixed over time.[9]

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[7]More generally, our wage bargaining setup relates to the competing-auction theories of McAfee (1993); Peters (1997); Peters and Severinov (1997).

[8]Changing the education threshold, for instance by defining as under-employed an individual with a college degree working in an occupation requiring "some college but no degree", gives very similar conclusions and stylized facts. Only the average level of under-employment is changed.

[9]Alternatively, we can define education requirements from the average education level observed in each occupation. As a robustness exercise, we construct the typical path of entry into an occupation from the average education level of that occupation during the 1984–1990 sample period, and we define the under-employment rate as the fraction of workers that are over-qualified relative

Figure 1 plots the US under-employment rate—the fraction of working college-educated individuals who are under-employed—, cleared from composition effects over 1983–2013.[10] We can see that under-employment is counter-cyclical, increasing in periods of slack labor market, but also that under-employment lags unemployment (by 5 to 8 quarters, Table 1).

1.2 The permanence of under-employment

We now show using CPS micro data that becoming under-employed is not a transitory state for high-educated workers. Instead, more than 70% of high-educated workers who move down the job ladder are still under-employed (or out-of-job) one year later.

In the CPS, an individual is surveyed for four consecutive months, left out for eight months, and then surveyed again for four consecutive months, allowing us to observe the same individual across 8 surveys. To evaluate the persistence of under-employment, we exploit the panel dimension of the CPS, and we measure the fraction of "newly under-employed" workers who are still under-employed one year later.

More specifically, we proceed in two steps. In a first step, we use the first four survey months to identify "newly under-employed" individuals. We define as "newly under-employed" an individual who (i) is unemployed in the first survey month, (ii) reports a previous occupation in line with his education level,[11] and (iii) finds an under-employed job during the first four survey months. In a second step, we use the four surveys conducted after the eight-month break to observe the worker's employment status exactly one year after becoming under-employed.

As shown in Table 2, only 28% of newly under-employed workers move back up the job ladder within a year, indicating that under-employment does not appear to

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to the typical path of entry. As apparent in Appendix Figure A1, the resulting under-employment series has very similar cyclical properties.

[10]We control for composition effects, as fluctuations in the under-employment rate could be due to demographics or industry-composition effects. For instance, if certain industries feature more under-employment than others, and if these industries have more cyclical employment, the under-employment rate would appear to be cyclical. To control for composition, we regress a dummy capturing whether an individual is under-employed on a set of industry fixed effects (defined at the 3-digit NAICS level), seasonal dummies, age and sex of the surveyed individual, state-fixed effects, and we plot the residual of this regression centered at the mean of the under-employment rate over the sample period. Because of changes in industry group definitions in 1983, the under-employment rate reported in Figure 1 only starts in 1983. As a sensitivity analysis, we also report in Appendix Figure A1 an under-employment rate series where we keep between-industry variations and only control for the observable characteristics of job seekers.

[11]The CPS asks job seekers about their previous occupation, which, combined with the reported education level, allows us to evaluate whether an individual was previously employed in an occupation matching his education level.

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be a strong jumping board for better jobs. Instead, under-employment appears to be a persistent state for many workers.[12]

1.3 The wage of under-employment

Using CPS micro data, we now evaluate the level and the cyclicality of the wage loss associated with becoming under-employed. We also assess whether high-educated workers earn a premium over low-educated workers employed in the same occupation.

The wage cost of under-employment To measure the wage cost of under-employment, we need to compare the hiring wage of under-employed individuals with the hiring wage of *identical* individuals who find occupations in line with their education (individuals referred to as "well-employed" hereafter). Unfortunately, this thought experiment is not possible because of unobserved heterogeneity: If individuals with lower (unobserved) ability select themselves into under-employment more often than individuals with higher (unobserved) ability, the hiring wage gap between under-employed and well-employed individuals will also reflect unobserved heterogeneity in ability instead of the true wage cost of under-employment.

In order to get as close as possible to the ideal thought experiment (within the constraints of the CPS) and evaluate the wage cost of under-employment, we proceed as follows.

We use data from the CPS Merged Outgoing Rotation Groups (MORG) over 1979–2013, and we consider the following baseline equation

$$\omega_{ijt} = \alpha + \beta D_{ijt}^{ue} + \gamma \mathbf{X}_{it} + \mu_t + \varepsilon_{ijt}, \qquad (1)$$

in which we model the hiring wage of a highly educated (some college or more) unemployed individual of type i finding a job of type j in period t. Our dependent variable, ω_{ijt}, is the logarithm of the real hourly wage of new hires.[13] D_{ijt}^{ue} is a dummy equal to 1 if the newly-hired individual is under-employed, and β is our coefficient of interest, meant to capture the "wage cost" of under-employment. The vector $\mathbf{X}_{it}$ includes observable individual characteristics, and μ_t is a time fixed effect to control for cyclical variations.

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[12]Assuming that labor market transitions follow a Markov process, this fraction implies that the average time to wait before moving up the job ladder for a newly under-employed worker is 1/0.28, which amounts to about 3.5 years. This finding is in line with independent evidence from Verhaest and Schatteman (2010) who follow school leavers for seven years after their entry into the labor market and find that under-employment is a persistent state.

[13]The hiring wage is the hourly wage deflated by the Personal Consumption Expenditures (PCE) deflator.

To make sure that the wage gap coefficient β is not driven by the presence of (unobserved) low-ability workers who are *always* under-employed, we estimate Equation (1) using only individuals who (i) are unemployed in a given survey month, (ii) report a previous occupation in line with their education level, and (iii) find a job—under-employed or not—during the next survey month.[14]

We then consider two specifications. In the first specification, we evaluate the wage gap after controlling for a range of observables $\mathbf{X}_{it}$ meant to capture selection into under-employment. Specifically, we include the usual observable characteristics (age, sex, state of residence) as well as previous occupation fixed effects. The use of previous-occupation fixed effects implies that we are comparing the hiring wage of individuals who used to work in the same occupation (defined at the 3-digit level).[15] By focusing on narrowly defined occupations, the goal is to restrict the sample to similar individuals. However, this first specification could still be affected by selection on unobservables since demographics and previous occupation fixed effects may not control for all of the unobserved heterogeneity across individuals. In a second specification, we thus add an additional control: the wage in the previous occupation.[16] Assuming that wage differences reflect unobserved heterogeneity (given that we already control for the usual observable characteristics), controlling for the previous wage will help us control for part of the unobserved selection into under-employment.

The first panel of Table 3 presents the results. As shown in the first column of Table 3, the unconditional hiring wage gap between under-employed and well-employed individuals is about 40 percent. Controlling for age, sex and previous occupation fixed effects reduces the estimate only slightly to 32 percent (second column). After including past wages, the estimate falls some more but remains large and significant at 28 percent (third column), implying that the wage cost of

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[14]Indeed, the group of under-employed workers is likely composed of workers with different unobserved characteristics. In particular, some low unobserved ability (or low college quality) college-educated workers may be "permanently" under-employed, i.e., always employed in jobs that do not require some college. Since these low-ability workers will always receive a lower wage than high-ability workers, looking at the unconditional wage difference between well-employed and under-employed workers would bias our result towards a high wage cost of under-employment. To address this issue, we only include "marginal" under-employed workers (workers who used to be well-employed and just moved down the job ladder) to the sample of newly-hired workers.

[15]There are around 500 occupations at the 3-digit level in the CPS, so that the use of previous occupation fixed effects allows us to compare individuals who used to work in similar occupations.

[16]Controlling for the wage in the previous occupation is however only possible for a subset of the sample. Since wages are only observed in the 4th and 8th survey, we need to (i) match individuals across the eight surveys (as in the previous section on the permanence of under-employment), (ii) isolate individuals who are well-employed (i.e., in an occupation matching their education level) in the 4th survey, and (iii) restrict ourselves to individuals who found a job between the 7th and 8th survey (since we are focusing on the hiring wage).

under-employment is likely to be substantial. Finally, controlling for unemployment duration, as a proxy for search behavior, does not modify the wage cost estimate.

Although we cannot evaluate the cyclicality of the wage cost of under-employment using our preferred specification that controls for the previous wage, we can still use our first specification with all the other controls (age, sex, state of residence, and previous occupation fixed effects).[17] As shown in Table 4, the wage cost of under-employment is higher in slack labor markets; being 4-5% larger whenever labor market slack (measured from the unemployment rate of the vacancy-unemployment ratio) is one-standard deviation above its mean. Note however that there are relative few wage observations in slack labor markets, and the evidence is only suggestive—being only significant at the 6 percent margin—.[18]

Wage premium   We now study the "wage premium" of under-employment, i.e., the difference between the wage of an under-employed high-educated worker and the wage of a low-educated worker employed in the *same* occupation, defined at the three-digit level.

We use as before data from the CPS MORG over 1979–2013, and we consider the following specification:

$$\omega_{ikt} = a + bD_{it}^c + c\mathbf{X}_{it} + \nu_k + d_t + e_{ikt}, \tag{2}$$

where i indexes an unemployed worker finding an occupation k requiring low education in period t, and D_{it}^c is a dummy equal to one if the worker has a college degree. In this specification, we control for invariant occupation-specific characteristics (fixed effects ν_k for three-digit occupational code). The coefficient b then captures the worker-specific wage premium associated with a college education.

As shown in the second panel of Table 3, we find that high-educated and low-educated workers hired in the same occupation are treated differently by firms: high-educated workers receive a premium in their hiring wage of about 25 percent

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[17]Because of sample size, we cannot assess the cyclicality of the wage cost of under-employment with the same restricted sample as in Table 3. We use instead the hiring wage of all under-employed workers.

[18]Despite all our efforts to control for changes in composition, some of the cyclicality in the wage cost of under-employment could be driven by cyclical variations in the relative (unobserved) ability of high-skilled in the two islands. With unobserved heterogeneity among the high-skilled (for instance, college degrees of different values), the relative ability of job seekers in the high-tech island would increase during a recession, if the low-ability high-skill workers are more likely to become under-employed than the high-ability high-skilled and this would widen the observed wage gap between the two islands. To allow for this possibility and make an apple-to-apple comparison between data and model, our model in Section 3 will feature (unobserved) worker heterogeneity among the high-skilled.

compared to low-educated workers.[19]

2 A static general equilibrium model of under-employment

In this paper, we develop a search model that can rationalize the existence and counter-cyclicality of under-employment. A novel aspect of the model is that matching is not strictly random. As we argue in the Appendix, explaining the counter-cyclicality of under-employment in a model with random matching requires that either the wage cost of under-employment is lower in recessions or that finding a job in a low-requirement occupation is easier in recessions. However, neither of these two conditions appear to hold in the data.

Instead, we propose a model with an alternative matching process, in which a vacancy can simultaneously receive multiple applications and where applicants compete for the job during the wage bargaining process. We will see that this setup endogenously generates a "ranking" mechanism à la Blanchard and Diamond (1994), in which high-skill applicants are systematically hired over less-skilled competing applicants.[20]

We first develop a static general equilibrium model of under-employment and describes its three key ingredients: (i) heterogeneity across workers and jobs, (ii) coordination frictions, and (iii) wage competition between workers. We leave a dynamic and stochastic version of the model for the next section.

2.1 Environment

We first describe the environment of the static model, in particular the matching friction and the wage bargaining setup.

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[19]This number is consistent with previous findings on the effect of over-education on wages (Duncan and Hoffman, 1981). Note however that unobserved heterogeneity in occupations could lead us to over-estimate the wage premium enjoyed by high-educated workers. For instance, even at the three-digit occupation level that we observe, high-educated workers could be employed in slightly more complex tasks. To provide some assurance that this is not the case, we exploit the overlap between the Census occupation classification and the SOC. The Census occupation classification is derived from the SOC but is also less detailed, as the SOC divides many Census occupations into sub-groups. To isolate more homogeneous groups, we estimate the same specification as in column (3) but using only the Census occupations that are not further disaggregated in the SOC. Column (4) shows that the estimated wage premium is almost unchanged (.236 instead of .251).

[20]In addition to being intuitively appealing, we show in the Appendix that the idea of ranking is consistent with experimental data compiled by Kroft, Lange and Notowidigdo (2013). Using fictitious job applications in which both the quality of individual applications and the quality of the application pool are randomized, we find that the rank of a candidate in a queue of applications influences his call-back rate beyond the sheer quality of his resume.

Preferences, technology and market structure There are two types of risk neutral agents in the economy, workers and firms, and the economy consists of two islands indexed by $j \in \{L, H\}$.[21]

Island H has a higher technology level than island L, so that firms operating in islands H are more productive and referred to as "high-tech". A firm consists of one vacancy, and a firm can enter an island j by posting a vacancy at a cost $c_j > 0$. The number of vacancies in each island, v_j, will be determined endogenously by firm entry.

Workers are similarly divided into two types $i = \{\ell, h\}$ characterized by different productivity levels. Workers of type h are the most productive and referred to as the high-skilled, while workers of type ℓ are referred to as the low-skilled. There is a mass n_h of type h agents and a mass n_ℓ, normalized to one, of type-ℓ agents. A worker with a job provides inelastically one unit of labor to the firm and receives a salary ω. A worker without a job receives 0.[22]

Denote by $q_L = \frac{n_\ell}{v_L}$ the ratio of type ℓ individuals to job openings in island L and by $q_H = \frac{n_h}{v_H}$ the ratio of type h individuals to job openings in island H. q_L and q_H can be seen as the "initial" queue lengths in each island for a given number of vacancies, i.e., the hypothetical queue lengths corresponding to the case where low-skill workers are in the low productivity island, high-skill workers in the high productivity island, and workers are not allowed to move.

A firm operating in island j and paired with a worker of type i produces φ_{ij} for $(i, j) \in \{\ell, h\} \times \{L, H\}$. Unmatched jobs and workers produce nothing.

Finally, in order to ensure assortative matching in equilibrium, we posit some complementarity between workers' skill and firms' technology and assume (log) super-modularity, i.e., $\varphi_{\ell H}/\varphi_{\ell L} < \varphi_{hH}/\varphi_{hL}$.

Coordination frictions Each worker decides to which island to send a job application.[23] In a large anonymous market, workers cannot coordinate on which firm to apply to, leading to coordination frictions in each island. Some firms will get multiple applications, while others receive none. Some firms will receive applications from workers of different types, while others will receive applications from workers

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[21]We focus on the case with only two islands, since that case already contains the most interesting lessons from our model. Characterizing the equilibrium in a model with more than two islands is relatively straightforward and is discussed in the online Appendix.

[22]The assumption of no unemployment benefits or home production is only used here for analytical simplicity and will be relaxed in the dynamic version of the model.

[23]An observationally equivalent formulation of our model would be to assume that each worker can simultaneously search across different islands but must choose how to allocate his (finite) search time across islands. In that formulation, a high-tech worker would send his application to an island at random, with the probability x_h of sending to a given island being the choice variable.

of the same type.

We assume that workers apply at random in a market with many workers and firms. We represent the matching process in each island by an urn-ball matching function as in Butters (1977), in which each application (ball) is randomly and independently allocated to a vacancy (urn). With a large number V of vacancies and numbers N_ℓ and N_h of low-skill and high-skill applicants, the probability $P(a_\ell, a_h)$ that a firm faces a_ℓ low-skill applicants and a_h high-skill applicants follows a multinomial distribution which can be approximated with a Poisson distribution,

$$P(a_\ell, a_h) = \frac{\left(\frac{N_\ell}{V}\right)^{a_\ell} e^{-\frac{N_\ell}{V}}}{a_\ell!} \frac{\left(\frac{N_h}{V}\right)^{a_h} e^{-\frac{N_h}{V}}}{a_h!}.$$

Wage negotiation and the hiring decision In this section, we describe the wage bargaining process that takes place between a firm and its (possibly multiple) job candidate(s). Specifically, we present a tractable bargaining setup that can capture wage negotiations with (i) multiple and (ii) heterogeneous applicants in a non-random hiring setting. The outcome of our wage bargaining collapses to the standard Nash bargaining outcome when the firm negotiates with only one applicant.

There is perfect information, and all agents observe the pool of applicants and their types. With probability $1 - \beta$, the firm makes a take-it-or-leave-it offer to its preferred candidate, and with probability β each candidate makes a take-it-or-leave-it offer to the firm, as in a first-price sealed-bid auction.

Let φ_{1st} denote the output generated by the first-best applicant (the most productive) and $\varphi_{2nd} \leq \varphi_{1st}$ denote the output generated by the second-best applicant (with the convention that $\varphi_{2nd} = 0$ if there is no second applicant). The outcome of the wage bargaining process is:[24]

$$\omega = \beta(\varphi_{1st} - \varphi_{2nd}). \tag{3}$$

In expectation, the firm gets all the surplus generated by the second-best applicant and a fraction $(1 - \beta)$ of the additional surplus generated by the first-best applicant over the second-best.[25]

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[24]To prove Equation (3), consider the two bargaining situations. When the firm makes the take-it-or-leave-it offer (with probability $1 - \beta$), it captures all the surplus: the first-best applicant is hired and only gets paid his outside option. When job applicants make the take-it-or-leave-it offer (with probability β), the first-best applicant gets the job by offering a wage that gives the firm all the surplus from the second-best applicant. Thus, even when he makes the offer, the first-best candidate only extracts his marginal surplus over the second-best candidate.

[25]Our simple wage bargaining game provides a tractable wage bargaining rule. Allowing for a more general bargaining game would complicate the analysis but would not affect the property that the outside option of the firm depends on the quality and number of the other applicants. For

Note that the outcome of our wage bargaining set-up encompasses some well-known allocation mechanisms. When there is only one applicant (i.e., $\varphi_{2nd} = 0$), the wage bargaining outcome is identical to the standard Nash bargaining case: the worker gets a wage $\omega = \beta\varphi_{1st}$, i.e., a share β of the surplus. When the two best candidates are identical (i.e., $\varphi_{1st} = \varphi_{2nd}$), as would be the case with homogeneous applicants, the wage bargaining outcome coincides with job auctions (Shimer, 1999; Julien, Kennes and King, 2000): the firm gets all the surplus from the match, and the hired candidate (chosen at random among the applicants) gets his outside option.

Timing The timing of events is as follows. (i) Each worker chooses which island to send an application to. In parallel, each potential firm entrant decides whether to post a vacancy in any given island; (ii) In each island, applications are randomly allocated to vacancies; (iii) A wage negotiation ensues between the firm and its (possibly multiple) applicants; (iv) Firm-worker matches are formed and production starts. Finally, firms pay workers and realize profits.

2.2 Equilibrium

2.2.1 Partial equilibrium with exogenous labor demand

In order to clarify how workers decide on which island to search, we start with the partial equilibrium (PE) with exogenous labor demand, i.e., taking the number of vacancies (and thus initial queue lengths (q_H, q_L)) in each island as given.

Definition 1. *Partial Equilibrium Allocation.*

Workers make optimal decisions. Each worker of each type $i \in \{\ell, h\}$ decides on which island $j \in \{L, H\}$ to search for a job. The equilibrium is given by a mapping of such worker choices $C_i : [0, n_i] \mapsto \{L, H\}$ for each type i.

Let x_h denote the under-employment rate, i.e., the share of type-h workers searching in island L, and let $E\omega_{i,j}$ denote the expected income of an individual of type i searching in island j.

The following proposition characterizes the equilibrium allocation in partial equilibrium:[26]

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instance, one could think of a more general framework in which the firm starts a game of alternating offers with the best applicant, and that, if these negotiations break-down, the firm starts a game of alternating offers with the second-best applicant, etc., as in models of intra-firm bargaining (Stole and Zwiebel, 1996; Brügemann, Gautier and Menzio, 2015). While more involved, the outcome of the negotiation would be qualitatively comparable: the best-applicant would get the job and his wage would depend negatively on the productivity and number of other applicants.

[26]We only focus on economies with a positive rate of under-employment in equilibrium. The online Appendix describes the condition that ensures the existence of a strictly positive rate of

Proposition 1. *There is a unique equilibrium allocation of workers satisfying:*

- *type-h workers are indifferent between islands L and H, and x_h is given by the arbitrage condition*

$$A(x_h) = -E\omega_{hH} + E\omega_{hL} = 0$$

  with

$$E\omega_{hH} = \beta e^{-q_H(1-x_h)}\varphi_{hH}, \quad E\omega_{hL} = \beta e^{-q_L x_h n_h}\left[e^{-q_L}\varphi_{hL} + (\varphi_{hL} - \varphi_{\ell L})\left(1 - e^{-q_L}\right)\right].$$

- *type-ℓ workers only look for jobs in island L and their expected income is*

$$E\omega_{\ell L} = \beta e^{-q_L(1+x_h n_h)}\varphi_{\ell L}$$

Proof. Online Appendix. □

Each worker searches for a job in the island that provides the highest expected wage, and in equilibrium, a high-skill (type-h) worker is indifferent between looking for a job in island H—the "high-tech island"—and looking for a job in island L—the "low-tech island"—, while a low-skill (type-ℓ) worker strictly prefers looking for a job in the low-tech island (island L). The arbitrage condition, $A(x_h) = 0$, determines the equilibrium allocation of type-h workers across the two islands.

The left panel of Figure 2 depicts the equilibrium allocation of high-skill workers as the intersection of the $E\omega_{hL}$ curve, the expected wage earned in island L, and the $E\omega_{hH}$ curve, the expected wage earned in island H. The $E\omega_{hH}$ curve is increasing in x_h: an increase in the fraction of high-skill workers searching in island L lowers congestion in island H, which lessens the competition high-skill workers face in island H and increases the expected wage. By contrast, an increase in the fraction of high-skill workers searching in island L makes island L more congested, which increases the competition workers face in island L and lowers the expected wage. Intuitively, the expected income of high-skill workers is driven by their uniqueness, as it determines both their ability to find a job easily (by being preferably hired over low-skill workers) and their ability to obtain a wage premium over low skill workers. As the number of high-skill workers increases, each high-skill worker becomes less unique and thus has a lower job finding rate (facing more competition from their peers) and receives a lower wage premium.

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under-employment and shows that the equilibrium is unique. The condition is that the number of vacancies in island H is not too large so that $E\omega_{hH} < E\omega_{hL}$ when $x_h = 0$.

2.2.2 Implications of under-employment
In this section, we discuss two novel implications of our model.

The trickle-down of unemployment First, under-employment can have regressive distributional implications. When high-skill workers move down the job ladder, they take the jobs of low-skill individuals. Through this process, unemployment trickles down from the upper-occupation groups to the lower-occupation groups.

To see this, consider an adverse labor demand shock affecting only the high-tech island, i.e., an increase in the queue length q_H. As shown in the left panel of Figure 3, it shifts down the $E\omega_{hH}$ curve—the expected wage earned in island H—, and generates a higher equilibrium under-employment rate x_h as high-skill workers use under-employment to smooth the shock and the associated decline in expected income. As a result, low-skill workers see a decrease in their expected earnings (as shown by the $E\omega_{\ell L}$ curve), because they face more competition from high-skill workers searching in island L. In other words, a shock affecting the high-type group trickles down to the lower-type group.

The counter-cyclicality of unemployment Second, aggregate shocks generate counter-cyclical movements in under-employment. To see this, consider an adverse aggregate shock that affects both islands equally. Specifically, the shock is such that, absent any change in the under-employment rate of high-skill workers, both worker types would be equally affected by the shock. Then, one can show that under-employment unambiguously increases. The following corollary formalizes this result:

Corollary 1. *Consider an adverse aggregate shock that affects the queue lengths in each island such that*

$$\Delta q_H(1 - x_h) = \Delta q_L(n_h x_h + 1) > 0$$

Then, the level of under-employment x_h increases with

$$\Delta x_h = \frac{\varphi_{hL} - \varphi_{\ell L}}{\varphi_{hH}(q_L n_h + q_H)} e^{-q_L x_h n_h + q_H(1-x_h)} \Delta q_L > 0.$$

Proof. Online Appendix. □

To help understand this result, the right panel of Figure 3 plots the corresponding experiment. As can be seen in Figure 3, while the aggregate shock shifts the wage curves $E[\omega_{hH}]$ and $E[\omega_{\ell L}]$ by the same amount, Corollary 1 states that the shift in the

$E[\omega_{hL}]$ curve is less pronounced than the shift in the $E[\omega_{hH}]$ curve. This reflects the fact that, because of the ranking advantage enjoyed by the high-skilled, they are less affected by the decrease in job openings in the low-tech island than by the decrease in job openings in the high-tech island. As a result, high-skill workers respond to the adverse aggregate shock by moving to the low-tech island in greater proportion, and under-employment unambiguously increases, i.e., under-employment is counter-cyclical.

Intuitively, our result comes from the fact that high-skill workers can partially jump the queue of unemployed in the low-tech island but *not* in the high-tech island.[27] As a result, the number of job seekers relative to job openings, i.e., the length of the unemployment queue, matters less in the low-tech island, and high-skill workers are less affected by an increase in the queue of unemployed in the low-tech island than by the same increase in the high-tech island. The high-skilled can thus smooth an adverse aggregate shock at the expense of the low-skilled by moving down the job ladder in greater proportion, and under-employment is counter-cyclical.

2.2.3 General Equilibrium with Endogenous Labor Demand We now characterize the general equilibrium (GE) with endogenous labor demand. There is an arbitrarily large mass of potential entrants who can settle in island j. A firm still consists of one vacancy, and a firm can enter an island j by posting a vacancy at a cost $c_j > 0$. With free entry, firms will enter in each island j until the point where expected profits, denoted by π_j, equal the fixed cost c_j. The number of firms and vacancies in each island will thus be determined endogenously by firm entry.

Definition 2. *General equilibrium allocation with endogenous firm entry.*

Workers and firms make optimal decisions. Each worker of type $i \in \{\ell, h\}$ decides on which island $j \in \{L, H\}$ to search for a job, and each potential firm entrant in island $j \in \{L, H\}$ decides whether or not to post a vacancy. The equilibrium is given by worker choices $C_i : [0, n_i] \mapsto \{L, H\}$ for each worker type i and firm choices $F_j : [0, \infty) \mapsto \{0, 1\}$ for each island j.

The following proposition characterizes the equilibrium allocation:

Proposition 2. *There is a unique equilibrium allocation satisfying*

- *The arbitrage conditions characterizing the allocation of workers:*

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[27]Indeed, high-skill workers do not benefit from any ranking advantage in the high-tech island, since no low-skill worker search in that island in equilibrium.

- *type-h workers are indifferent between islands L and H, and x_h, the share of type-h workers searching in island L, is given by the arbitrage condition*

$$A(x_h, q_L) = -E\omega_{hH}(x_h, q_H) + E\omega_{hL}(x_h, q_L) = 0.$$

- *type-ℓ workers only look for jobs in island L.*

- *Firms' free entry conditions (market clearing) in islands L and H*

$$\begin{cases} \pi_L(x_h, q_L) = c_L \\ \pi_H(x_h, q_H) = c_H \end{cases}$$

Proof. Online Appendix. $\square$

To depict the GE allocation graphically, the right panel of Figure 2 proceeds similarly to the left panel in the PE case and identifies the equilibrium under-employment rate as the intersection of the $E\omega_{hL}$ curve, the expected wage for the high-skilled in island L, and the $E\omega_{hH}$ curve, the expected wage for the high-skilled in island H.[28]

Contrasting the left and right panels of Figure 2 reveals two differences between the PE and the GE allocation. First, the $E\omega_{hH}$ curve, the expected wage earned in island H, is now flat and no longer upward slopping as in PE. This result illustrates how "matching with ranking" reduces to random matching (as in Pissarides, 1985) when workers are homogeneous. Indeed, in our model the high-tech island is homogeneous in equilibrium and only populated by high-skill job seekers. In Pissarides (1985), as in our model, the supply of homogeneous labor has no effect on the equilibrium queue length.[29] Second, the $E\omega_{hL}$ curve is decreasing in x_h, as in the PE case, but the curve is now less steep. This occurs because of a general equilibrium job creation effect, in which firms respond to changes in the average productivity of the unemployment pool. Specifically, an increase in the share of high-skill workers searching in island L raises firms' probability to meet high-skill applicants (who generate a higher surplus than low skill applicants), raises firms' profits, and thus leads to more job creation.[30]

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[28]The proofs underlying Figure 2 are in the online Appendix.

[29]An increase in the number of job seekers raises firms' matching probability, i.e., reduces hiring costs, which leads more firms to enter the market, so that profit and thus the queue length are ultimately unchanged. In other words, job creation always compensates any change in the number of job seekers in a homogeneous island.

[30]Such a "job creation" GE effect is well known in random search models with heterogeneous workers. See e.g., Albrecht and Vroman (2002); Charlot and Decreuse (2010). However, in a model with random matching, the job creation effect leads to an upward sloping wage schedule, so that low skill workers benefit from under-employment. In our case, the job creation effect does not lead to an upward sloping wage schedule but makes the wage schedule less downward sloping.

With the slight differences between PE and GE in mind, it is easy to see that the redistributive implications of under-employment carry on from the PE case to the GE case. As in the PE case, an adverse labor demand shock affecting island H (a downward shift in the $E\omega_{hH}$ curve) will trickle down to island L, as high-skill workers respond to higher congestion in the high-tech island by increasing their under-employment rate, which leads to higher congestion for low-skill workers. The only difference with respect to the PE case, is that, thanks to the increased number of high-skill workers in the low-tech island, job creation is higher in the low-tech island, and this GE effect dampens the crowding-out coming from the larger number of high-skill workers searching in the low-tech island.

2.3 Constrained optimal allocation

We now consider the efficiency of the decentralized allocation. We study the problem of a planner who can only allocate workers across islands in order to maximize total output (net of the cost of posting vacancies) subject to firms' free entry condition and subject to hiring frictions in each island.[31]

Proposition 3. *The constrained optimal allocation (x_h^*, q_L^*) is characterized by the firms' free entry conditions in islands L and H, and*

$$
\begin{aligned}
A(x_h^*, q_L^*) &= -E\omega_{hH} + E\omega_{hL} \\
&= (1-\beta)\, n_h q_L^* \varphi_{\ell L} e^{-q_L^*(2x_h^* n_h + 1)} \left(\varphi_{hL} - \varphi_{\ell L}\right) \frac{1}{\frac{\partial \pi_L(x_h^*, q_L^*)}{\partial q_L}} \\
&\geq 0
\end{aligned}
\tag{4}
$$

with the expression for $\frac{\partial \pi_L(x_h, q_L)}{\partial q_L} > 0$ given in the online Appendix.

If $\beta < 1$ and $\varphi_{hL} - \varphi_{\ell L} > 0$, the decentralized allocation (x_h, q_L) is inefficient and has too much under-employment: $x_h > x_h^*$.

Proof. Online Appendix. □

The proposition states that the decentralized allocation is, in general, not efficient, and that there is too much under-employment. While we leave a more detailed discussion of efficiency for the online Appendix, we now discuss the intuition behind this result.

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[31]Note that our exercise differs from the usual approach in the search literature (Hosios, 1990), since the planner cannot allocate vacancies across islands. The usual trading externalities are also present in our framework, but we abstract from them to focus on the worker allocation problem. An extension of our efficiency discussion would be to consider the problem of a planner allocating *both* workers and vacancies across islands.

As a preliminary step, it is helpful to note that, in this worker allocation problem, maximizing total output while satisfying firms' zero profit condition is the same as maximizing total labor income. Thus, when discussing externalities, we can focus on the effect of a worker's decisions on the expected wage of other job seekers.

Consider first island H; the high-tech island. An increase in the share of high-skill workers in the low-tech island lowers competition between workers in the high-tech island, which ceteris paribus raises the expected wage of job seekers. At the same time, the lower number of job seekers in the high-tech island increases hiring costs for firms, which leads to less job creation and ceteris paribus lowers the expected wage of job seekers. As we saw in Section 2.2.3, when workers are homogeneous, the two effects exactly compensate each other: job creation adjusts to movements in the number of job seekers, and the expected income of high-skill workers is independent of the under-employment rate. In other words, high-skill workers exert on net no externalities on the high-tech island.

Consider now island L; the low-tech island. Proposition 3 states that there are too many high-skill job seekers in the low-tech island. Intuitively, the inefficiency stems from the fact that the marginal high-skill job seeker in island L can change how the surplus of a match gets shared (on average) between firms and workers. To see this, note that in our wage bargaining setup, a high-skill worker receives on average a share β of his marginal surplus over the second-best applicant—Equation (3)—, so that a share $1 - \beta$ of this marginal surplus is given to the firm. When $\beta = 1$ the high-skill worker fully captures his expected marginal surplus, so that his actions do not distort the allocation, and the decentralized allocation is efficient. However, when $\beta < 1$, the firm receives some of that marginal surplus, so that a marginal high-skill job seeker in island L will increase the (average) share of the surplus going to the firm. Firms respond by posting more vacancies than they would have if an average job seeker had entered island L, this makes island L too attractive to high-skill workers, and there is too much under-employment.[32]

The central difference between the low-tech and the high-tech island is that workers are not homogeneous in the low-tech island, and that a marginal high-skill

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[32]The inefficiency result of Proposition 3 is thus tied to the wage bargaining game and to the sharing rule between the firm and the first-best applicant in the presence of a second-best applicant. In that wage bargaining configuration, one can see the surplus of the (hired) first-best applicant as being made of two parts: a first part being the surplus generated by the second-best applicant and a second part being the marginal surplus generated by the first-best applicant over the second-best. Importantly, these two parts are shared differently between the firm and the first-best applicant: while the surplus of the second-best applicant is fully captured by the firm, the marginal surplus of the first-best applicant is split between the firm and the worker. The inefficiency stems from this property, because it implies that the share of high-skill job seekers affects the average share of the surplus going to the firm.

worker in island L changes the composition of the pool of job seekers, which can alter the surplus sharing rule between firms and workers and distort the allocation. When the marginal high-skill worker gets paid his marginal surplus and thus does not affect the firm surplus as in job auctions (Shimer, 1999; Julien, Kennes and King, 2000)—which would correspond in our setup to $\beta = 1$—, the decentralized allocation is constrained efficient. The inefficiency also disappears when the heterogeneity among applicants goes away: when $\varphi_{hL} - \varphi_{\ell L} \to 0$ the low-tech island is populated by a quasi-homogeneous population of workers, and the decentralized allocation is efficient as shown by Equation (4). In a similar vein, when high-skill workers are very few ($n_h \approx 0$) and can easily "dilute" themselves in the low-tech island, most bargaining configurations will be between low-skill applicants, high-skill job seekers have little effect on surplus sharing, and Equation (4) shows that the inefficiency disappears. In the other polar case where there are infinitely more high-skill workers than low-skill workers ($n_h x_h >> 1$), most bargaining configurations will be between high-skill applicants and the inefficiency disappears again.

3 A quantitative dynamic stochastic model with under-employment

We now present a dynamic and stochastic version of our general equilibrium model. We calibrate the model, evaluate its ability to match the key facts documented in the empirical section, and then use it to assess the magnitude of the distributional consequences of under-employment, and the cyclicality of the inefficiency associated with under-employment.

3.1 A dynamic stochastic general equilibrium model

To bring our theoretical framework to the data, we extend our static general equilibrium model in a number of dimensions.

First, we extend the model to a (i) dynamic and (ii) stochastic setting, where the economy is hit by aggregate technology shocks. Second, we introduce unobserved worker heterogeneity within worker groups.

The economy is composed of a continuum of infinitely lived workers and a continuum of infinitely lived firms. Time is discrete. All agents are risk-neutral and let δ denote the discount factor. As in the static model, there are two types of firms located in two distinct islands indexed by j. Each firm can have at most one job, and a vacancy can freely be created in each period subject to an island-specific vacancy posting cost c_j. Similarly, workers are heterogeneous and can be of different types i.

However, and different from the static model, we introduce unobserved hetero-

geneity among high-skill workers.[33] Specifically, we allow for two (unobservable) type h workers; a low-quality type $\underline{h}$ and a high-quality type $\overline{h}$ with productivity in island j satisfying $\varphi_{\underline{h}j} < \varphi_{\overline{h}j}$. Intuitively, not all high-educated workers are equally skilled; some are intrinsically smarter than others. If the smarter college graduates never become under-employed, the econometrician (as in Section 1, Table 3) will over-estimate the true wage cost of under-employment. Introducing unobserved worker heterogeneity allows us to capture this phenomenon in the model and thus allows for a cleaner comparison with our empirical findings from Section 1.[34] While these two sub-types are unobservable to the econometrician, they are perfectly known to workers and firms. We denote by η_h the ratio of type-$\overline{h}$ to type-$\underline{h}$ workers.

Also different from the static model, the productivity of a match is now stochastic with the log-productivity $\ln \varphi_{ij,t}$ in island $j \in \{L, H\}$ for individual $i \in \{\ell, \underline{h}, \overline{h}\}$ following an AR(1) process

$$\ln \varphi_{ij,t} = \rho \ln \varphi_{ij,t-1} + (1 - \rho) \ln \varphi_{ij} + \tau_j \varepsilon_t,$$

with $\rho < 1$, φ_{ij} the average productivity level, ε_t an aggregate productivity shock affecting all islands and τ_j the elasticity of island j's productivity with respect to aggregate shocks. The parameter τ_j allows us to capture the fact that some occupations may be more cyclical than others.

Within each period, the timing of actions is the following. The state of nature is revealed, and production takes place. After production, a share s of firm-worker matches is destroyed exogenously. Vacancies are posted, and job seekers decide in which island to search. Upon matching, firms and applicants negotiate over the wage and production starts the next period.

Denote by m a wage bargaining configuration (e.g., a firm bargaining with one high-skill applicant and one low-skill applicant), by $p_{ij,t}^w(m)$ the probability for a worker type-i in island j to be in wage bargaining configuration m, and by $p_{j,t}^f(m)$

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[33]Given that our economy only features two islands, the low-skill group cannot react to aggregate shocks by moving further down the job ladder, and introducing unobserved heterogeneity among the low-skill group is of limited interest. See the online Appendix for a discussion of an economy with three islands, in which mid-skilled workers can smooth shocks (and react to a higher number of high-skill job seekers) by themselves moving to a lower-requirement island.

[34]More specifically, introducing unobserved heterogeneity is necessary to rationalize the existence of under-employment given the large *observed* wage loss associated with under-employment (about 28 percent, see Table 3). Echoing a point previously made by Hornstein, Krusell and Violante (2011), the wage loss associated with under-employment must be compensated by an increase in job finding probability, which is difficult to rationalize given the already high job finding rates observed in US data. In our model, only the low-quality high-skill workers become under-employed in equilibrium, so that the actual wage cost of under-employment is smaller than the wage cost observed by the econometrician.

the probability for a firm in island j to be in wage bargaining configuration m. $U_{ij,t}$, the value of unemployment for a type-i worker in island j is then

$$U_{ij,t} = b + \delta \left[\sum_m p_{ij,t}^w(m) E_t W_{ij,t+1}(m) + \left(1 - \sum_m p_{ij,t}^w(m) \right) E_t U_{ij,t+1} \right],$$

and $\Pi_{j,t}^o$ the value for a firm to open a vacancy in island j is

$$\Pi_{j,t}^o = \delta \sum_m p_{j,t}^f(m) E_t \Pi_{j,t+1}(m),$$

with b the flow value of leisure, $E_t W_{ij,t+1}(m)$ the expected value of employment for a type-i worker in island j after facing the bargaining configuration m, and $E_t \Pi_{j,t+1}(m)$ the corresponding expected value for the firm.

Upon matching, surplus sharing is determined using the same bargaining game as in the static model. The present discounted surplus of the first-best applicant over the second-best is thus split in shares β and $1 - \beta$ between the firm and the (hired) first-best applicant, while the firm additionally gets all the present discounted surplus from the second-best applicant. To express this surplus sharing rule, we can use a representation in terms of flow payments. Letting $b_{i,t} = (1 - \delta) U_{i,t}$ denote the reservation wage of a type-i worker, the outcome of the wage bargaining process for a match in island j with initial bargaining configuration m can be written as

$$\begin{cases} w_{ij,t}(m) = b_{1^{st}(m),t} + \beta \left[\left(\varphi_{1^{st}(m)j,t} - b_{1^{st}(m),t} \right) - \left(\varphi_{2^{nd}(m)j,t} - b_{2^{nd}(m),t} \right) \right] \\ \pi_{j,t}(m) = (1 - \beta) \left[\left(\varphi_{1^{st}(m)j,t} - b_{1^{st}(m),t} \right) - \left(\varphi_{2^{nd}(m)j,t} - b_{2^{nd}(m)j,t} \right) \right] \end{cases}$$

where $w_{ij,t}(m)$ and $\pi_{j,t}(m)$ are the flow payments going to the worker and the firm, with i denoting the hired applicant and $1^{st}(m)$ (resp. $2^{nd}(m)$) denoting the type of the first-best applicant (resp. second-best applicant) in bargaining configuration m.[35]

Finally, the value functions $W_{ij,t}(m)$ and $\Pi_{j,t}(m)$ satisfy

$$\begin{cases} W_{ij,t}(m) = w_{ij,t}(m) + \delta(1 - s) E_t W_{ij,t+1}(m) + s\delta E_t U_{ij,t+1} \\ \Pi_{j,t}(m) = \pi_{j,t}(m) + \delta(1 - s) E_t \Pi_{j,t+1}(m) \end{cases}$$

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[35]An implicit assumption in this dynamic model is that the surplus sharing rule agreed upon in the initial bargaining state will not be renegotiated at future dates. This is a mild assumption, because the firm can credibly commit to never renegotiate with its worker at future dates. Indeed, it is never in the firm's interest to renegotiate, because bargaining with only one applicant (as would be the case in a renegotiation with the one hired worker) gives the firm its smallest share of the surplus (among all the possible initial bargaining configurations). Since both the firm and the worker receive a non-negative value from the match, it is never in either party's interest to break the match.

We can now characterize the equilibrium allocation of job seekers and vacancies in the following proposition, which generalizes Proposition 2 to a dynamic setting.

Proposition 4. *There is a unique equilibrium allocation of job seekers and vacancies satisfying the following conditions in each period t:*

- *type-$\overline{h}$ workers only look for jobs in island H,*

- *type-$\underline{h}$ workers are indifferent between islands L and H, and $x_{h,t}$, the share of type-$\underline{h}$ workers searching in island L, is given by the arbitrage condition*

$$A(x_{h,t}, q_{L,t}, q_{H,t}) = -U_{\underline{h}H,t}(x_{h,t}, q_{H,t}) + U_{\underline{h}L,t}(x_{h,t}, q_{L,t}) = 0$$

- *type ℓ workers only look for jobs in island L.*

The firms' free entry conditions are verified in islands L and H:

$$\begin{cases} \Pi_L^o(x_{h,t}, q_{L,t}) = c_L \\ \Pi_H^o(x_{h,t}, q_{H,t}) = c_H \end{cases}$$

Proof. Online Appendix. □

3.2 Calibration

To calibrate the model, we proceed as follows. We set values for the parameters that are directly observable, and we estimate the leftover parameters by matching key first moments of the steady-state economy. Table 5 lists all the parameter values used in the quantitative exercise.

We set a monthly discount factor corresponding to a risk-free interest rate of 5% per annum. We set workers' separation rate s and the ratio n_h of college-educated workers to non-college educated workers from CPS micro data over 1983–2013. We set the income replacement rate to 20%, and the workers' bargaining weight to 0.5, implying that workers and firms are equally likely to make an offer.

To use the urn-ball matching function in a quantitative context, we proceed as Blanchard and Diamond (1994) and introduce a matching efficiency term. Specifically, we assume that workers send out an application with an island-specific probability ν_j and we set the ν_j parameters by matching f_h and f_ℓ, the observed average job finding rates of high-skill and low-skill workers, which satisfy

$$\begin{cases} f_h = \left[(1 - x_h) f_{\overline{h}H} + x_h f_{\underline{h}L} + \eta_h f_{\overline{h}H} \right] / (1 + \eta_h) \\ f_\ell = f_{\ell L} \end{cases} \tag{5}$$

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where $f_{\bar{h}H}$, $f_{\underline{h}H}$, $f_{\underline{h}L}$ and $f_{\ell L}$ are the unobserved job finding rates of the different worker types in the different islands.[36]

To set τ_j, the cyclical elasticity of island j, we use job openings data by occupation from the Conference Board Help Wanted OnLine (HWOL) dataset. While job openings by degree-requirements are not observed, there is a close correspondence between degree requirements and high-level occupation groups: Services and professional occupations require a bachelor's degree in most cases (it is true for more than 90 percent of the 3-digit occupations in those two categories), while construction and sales typically require less than a high school degree (it is true for more than 75 percent of the 3-digit occupations in those two categories). We thus proxy job openings with high degree-requirements with job openings in services and professional occupations, and we proxy job openings with low degree-requirements with job openings in construction and sales. Since job openings are 20% less volatile in services and professional occupations than in construction and sales, we normalize $\tau_L = 1$ and set $\tau_H = 0.8$.

The model still requires parameter values for the output of each type of match ($\varphi_{\ell L}$, $\varphi_{\underline{h}L}$, $\varphi_{\bar{h}H}$, $\varphi_{\underline{h}H}$), the ratio η_h of high-quality to low-quality type-h workers, and the vacancy posting costs in island L and H (c_L and c_H). After normalizing the average level of $\varphi_{\underline{h}H}$ to one, we are left with six free parameters. Since there is no direct observable counterpart to these parameters, we estimate their values by matching the first moment of the following series: the under-employment rate (UE),[37] the observed wage cost of under-employment (w_{hH}/w_{hL}), the wage premium earned by over-educated workers ($w_{hL}/w_{\ell L}$)—all documented in Section 1—, the wage ratio between type-h and type-ℓ workers (w_h/w_ℓ) and the HWOL job openings to job seekers ratio in typical college-level occupations (q_H) and in typical high-school level occupations (q_L).

Table 6 reports the model values for the main steady-state moments: the under-

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[36]From the urn-ball matching function, we get that $f_{\bar{h}H}$, $f_{\underline{h}H}$, $f_{\underline{h}L}$ and $f_{\ell L}$ are given by

$$\begin{cases} f_{\bar{h}H} = \nu_H f(\nu_H q_H \eta_h) \\ f_{\underline{h}H} = \nu_H e^{-\nu_H q_H \eta_h} f(\nu_H q_H (1-x_h)) \\ f_{\underline{h}L} = \nu_L f(\nu_L q_L n_h x_h) \\ f_{\ell L} = \nu_L e^{-\nu_L q_L n_h x_h} f(\nu_L q_L) \end{cases} \tag{6}$$

with $f(q) = \frac{1-e^{-q}}{q}$ and ν_j the matching efficiency of the labor market in island j. Note that $f_{\bar{h}H}$, $f_{\underline{h}H}$, $f_{\underline{h}L}$ are unobservable for two reasons; first, we do not observe where high-educated workers are searching (i.e., in island L or in island H) so that we cannot compute their job finding rates, and second we do not observe whether a high-educated worker is a high-type $\bar{h}$ or a low-type $\underline{h}$.

[37]There is a difference between x_h, the (unobserved) fraction of high-skill workers *searching* in the low-tech island, and the observed under-employment rate, UE, which is the fraction of high-skill workers *employed* in the low-tech island and depends upon past flows and relative job finding rates. We specify the link between the two concepts in the online Appendix.

employment rate, the average job finding rates the different types, and the wage premiums across occupations and type. Panel A reports the target moments, and panel B reports unobserved moments. When studying Table 6, keep in mind our notation for the wages: w_{hH}, w_{hL} and $w_{\ell L}$ denote the *observed* wages of respectively type-h workers in island H, type-h workers in island L and type-ℓ workers in island L, while $w_{\underline{h}H}$, $w_{\underline{h}L}$ and $w_{\overline{h}H}$ denote the *unobserved* wages of respectively type-$\underline{h}$ in island H, type-$\underline{h}$ in island L and type-$\overline{h}$ in island H. Similar notations apply to the job finding rates.

Of particular interest are the unobserved wages and job finding rates that the parametrized model allows us to infer. First, under the parameter values of Table 5, the *actual* (unobserved) wage cost of under-employment ($w_{\underline{h}H}/w_{\underline{h}L}$) is 10 percent, instead of the observed 28 percent (w_{hH}/w_{hL}). With that wage cost of under-employment, the model can rationalize the high level of under-employment, because the cumulative wage loss over the expected duration of an under-employment spell is comparable to the cumulative income loss over the expected duration of an unemployment spell in the high-tech island.

As in the data, type-h workers in island L earn a wage premium over low skill workers of about 25% ($w_{hL}/w_{\ell L} = 1.6/1.28 = 1.25$, panel A).

Turning to job finding rates, the observed job finding rates of high- and low-skill workers are of the same order of magnitude (panel A), but they mask large differences across islands and unobserved types. Thanks to their ranking advantage over type-ℓ workers, type-$\underline{h}$ workers are about twice as likely to find a job when they search in the low-tech island L than in the high-tech island H ($f_{\underline{h}L} \approx 2f_{\underline{h}H}$, panel B). Type-$\overline{h}$ workers benefit from a similar ranking advantage over type-$\underline{h}$ workers in island H and are about twice as likely to receive an offer as their low-quality high-skill peers ($f_{\overline{h}H} \approx 2f_{\underline{h}H}$, panel B).

3.3   Model performance

We now assess the performance of our model in matching the key facts highlighted in Section 1. To solve the model and simulate the effect of aggregate shocks, the stochastic process for productivity is discretized using the Tauchen (1986) method. We solve the dynamic problem through value function iteration.

Impulse response functions   To illustrate the dynamics of the model, we first plot the impulse responses of the variables of interest to a negative aggregate productivity shock that lowers productivity in both islands. With lower productivity, job openings (i.e., the "initial" queue lengths q_L and q_H) decline in both islands

(top-right panel) and x_h increases as high-skill workers search in the low-tech island in greater proportion (bottom-left panel).[38]

Following an adverse aggregate shock, $f_{\underline{h}L}$ declines less than $f_{\underline{h}H}$, i.e., the job finding rate of type-$\underline{h}$ workers declines less in the low-tech island than in the high-tech island. As in Corollary 1 in the static model, this result comes from the ranking advantage enjoyed by type-$\underline{h}$ workers over type-ℓ workers. Because high-skill workers can partially jump the queue of unemployed in the low-tech island but *not* in the high-tech island, the length of the unemployment queue matters less in the low-tech island, and high-skill workers are less affected by an increase in the queue of unemployed in the low-tech island than by the same increase in the high-tech island. The high-skilled can thus smooth an adverse aggregate shock at the expense of the low-skilled by moving down the job ladder in greater proportion. As type-$\underline{h}$ workers become under-employed in greater proportion, they crowd out the type-ℓ workers, who see their job finding rate ($f_{\ell L}$) decline substantially more (middle-right panel).

Interestingly, a similar mechanism takes place in the high-tech island, which is also heterogeneous, and $f_{\overline{h}H}$ declines less than $f_{\underline{h}H}$ (middle-left panel). Type-$\overline{h}$ workers suffer a smaller decline in their job finding rate than the lower-skilled type-$\underline{h}$, because they enjoy a ranking advantage over type-$\underline{h}$ workers and are thus less affected by the adverse aggregate shock.

Finally, the wage cost of under-employment ($w_{\underline{h}H}/w_{\underline{h}L}$) goes up at the same time as the share of type-$\underline{h}$ searching in island L (bottom row).

Simulation We simulate an economy hit by aggregate productivity shocks. We simulate 32 years of monthly data (the same sample size as in the actual data used in Section 1), and we repeat the exercise 1,000 times to obtain key moments of the simulated data. All the model statistics are then aggregated to quarterly frequency, following the same procedure that we apply to the data. The volatility and auto-correlation of productivity are set to match the standard-deviation and auto-correlation of unemployment over 1979–2013.

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[38]Productivity declines more in the low-tech island than in the high tech island (top-left panel), because the low-tech island is more cyclical than the high-tech one ($\tau_L > \tau_H$). Regarding the relative responses of q_L and q_H, recall that the responses of job openings depend on two factors: (i) the elasticity of job openings to productivity changes (for a constant composition of the pool of job seekers), and (ii) the elasticity of job openings to changes in the composition of the unemployment pool as x_h changes. In our calibration, the estimated vacancy posting cost in the low-tech island is significantly lower than in the high-tech island, i.e., the value of unemployment is closer to the value of employment in the low-tech island, and this makes vacancy posting in island L more sensitive to productivity changes (echoing the point made by Hagedorn and Manovskii, 2008). Thus, absent movements in x_h, q_L would increase more than q_H. However, the increase in x_h stimulates job creation in island L and thus attenuates the initial effect of lower productivity, so that overall the impulse responses of q_L and q_H are of similar magnitudes.

Table 7 reports key moments of our simulated data and contrasts them with the data and the empirical regularities documented in Section 1. We report (i) the standard-deviation, (ii) the correlation with the unemployment rate and (iii) the serial correlation of five key variables: UE—the under-employment rate—, f_h and f_ℓ —the job finding rates of type-ℓ and type-h workers—, w_{hH}/w_{hL}—the (observed) wage cost of under-employment—, and $w_{hH}/w_{\ell L}$—the ratio between the wage of type-h workers in island H and the wage of type-ℓ workers in island L—.

Overall, the model does a very good job at matching the main features of under-employment documented in Section 1. First, with aggregate productivity shocks alone, the model generates counter-cyclical under-employment, as in the data, with a variance in line with that observed in the US (first column of Table 7).

Second, with a counter-cyclical under-employment rate, high-skill workers smooth aggregate shocks and therefore smooth fluctuations in their job finding rate. As a result, the job finding rate of high-skill workers is less volatile than the job finding rate of low-skill workers, as in the data (columns 2 and 3 of Table 7).

Third, the model generates the right cyclicality for both the wage loss associated with under-employment (w_{hH}/w_{hL}) and the wage differential across islands ($w_{hH}/w_{\ell L}$). As in the data, the wage loss is counter-cyclical, so that both the wage cost of under-employment *and* under-employment increase during recessions.[39] The intuition for the mechanism at play in our model is simple: as under-employment increases following a negative aggregate shock, type-h workers searching in island L become less unique, and competition with their peers becomes more intense. As a result, they extract a smaller share of the match surplus in island L, and the wage cost of under-employment w_{hH}/w_{hL} increases.[40]

Finally, the model does a good job at capturing the dynamics of labor market variables and notably the dynamics of under-employment. The model can capture the serial correlation of the main labor market variables (bottom row of Table 7) and does a good job with the lead-lag relationship between under-employment and unemployment (Figure 5). In fact, the model goes some way in reproducing the lagging behavior of under-employment, with a peak correlation around 5 quarter lags (versus 8 in the data). To understand this result, the key is to note that the stock of

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[39]Note that the wage correlations and relative volatilities are somewhat larger in the model than in the data. This is likely due to the substantial measurement error in the CPS wage data that inflates the variance of measured wages and leads to a downward bias in the estimated empirical correlations (for classical measurement error).

[40]The model also matches the relative volatility of w_h and w_ℓ, the observed wages of type-h and type-ℓ workers. In the data, we have $\sigma(w_h)/\sigma(w_\ell) = 1.56$, while the model generates $\sigma(w_h)/\sigma(w_\ell) = 1.45$. In the model, the observed wage of type-h workers is more volatile, because type-h workers adjust their under-employment rate along the cycle; trading-off lower wages for higher job finding rates in recessions, and vice-versa in expansions.

under-employed workers is a slow-moving variable, because the under-employment rate is a measure of the composition of the employment pool. Since the probability of employment separation is relatively low, the composition of the employment pool changes slowly, and the under-employment rate adjusts slowly to changes in the composition of the pool of job seekers. This slow adjustment of the under-employment rate is in contrast with the rapid adjustment of the unemployment rate. The unemployment rate adjusts fast, because the speed of adjustment of unemployment is driven by the job finding rate, which is an order of magnitude larger than the job separation rate (Shimer, 2005b; Barnichon and Nekarda, 2012).

3.4   Counterfactual experiments

In this final section, we use our calibrated model to (i) study the redistributive consequences of under-employment, and (ii) study the inefficiency associated with under-employment, notably its cyclical properties.

In a first experiment, we explore to what extent under-employment helps the high-skilled smooth business cycle fluctuations at the expense of the low-skilled. We perform a counterfactual simulation in which the shocks are identical to the previous simulation but in which we force the fraction of type-h workers searching in island L to remain constant over time.

Table 8 shows how holding x_h fixed affects the volatility experienced by low-skill workers. We can see that with a fixed under-employment rate, low-skill workers experience relatively smaller fluctuations in their job finding rate and in their expected wage. Note however that this redistributive effect of under-employment appears to be limited, as the decline in volatility is only in the order of 5 percent. The reason for this limited effect is a powerful general equilibrium effect. As high-skill workers search in the low-tech island in greater proportion, they stimulate job creation, which then dampens the initial crowding-out effect of having more high-skill job seekers. Absent any job creation effect (i.e., holding the number of job openings fixed), we found that the counter-cyclicality of under-employment would increase the volatility experienced by low-skill workers by 41 percent.[41] In other words, the job creation effect plays a critical role in dampening the negative effects of under-employment for low-skill workers. If that effect was not instantaneous or was weaker (for instance

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[41]To put the large compensating effect of job creation into perspective, recall that in a homogeneous labor market (as in Pissarides, 1985), the general equilibrium job creation effect is so strong that it *entirely* compensates for the increased congestion caused by having one more job seeker, as vacancy posting adjusts to the number of job seekers to keep the vacancy-unemployment ratio constant. In our model with heterogeneous job seekers, the job creation effect compensates about $100 - 5/41 \approx 87$ percent of the extra congestion created by an increase in x_h.

if firms could only imperfectly observe the composition of the unemployment pool), the redistributive effects of under-employment could be large.

In a second experiment, we consider an economy hit by the same sequences of shocks as before, and we determine what the planner would do if he could allocate workers optimally between islands.[42] Table 8 reports the key moments of this simulated constrained efficient allocation.

Confirming the analytical results of Section 2, the efficient allocation has a smaller average under-employment rate than the decentralized one, i.e., there is too much under-employment in the decentralized allocation.

Regarding the cyclicality of the inefficiency, we find that the inefficiency is largest in recessions, and the planner wants an even lower under-employment rate in recessions. In other words, a policy aimed at alleviating the consequences of under-employment would need to be counter-cyclical; providing a stronger incentive for high-skill workers not to move down the job ladder when jobs are scarce. To get some intuition behind this result, note that the inefficiency stems from the bargaining configuration in which the firm negotiates with one high-skill applicant and at least one other low-skill applicant. Indeed, it is precisely in that configuration that a marginal high-skill job seeker alters the average surplus sharing between firms and workers. In all the other configurations—bargaining with only one applicant or bargaining with two homogeneous applicants—, a marginal high-skill worker does not affect the surplus sharing rule. The inefficiency gets worse in recessions, because the probability of bargaining with heterogeneous applicants—the source of the inefficiency—increases in recessions. Indeed, ceteris paribus, as the number of job openings per job seekers decreases and the queue of job seekers increases, the probability that this queue includes one high-skill worker increases,[43] and so does the probability of a bargaining configuration with heterogeneous applicants.

4 Conclusion

We study empirically the phenomenon of under-employment in the US and show that (i) under-employment is strongly counter-cyclical, (ii) under-employment is costly—an under-employed worker earning substantially less than his non-under-employed counterpart—and the wage cost is counter-cyclical, and (iii) under-employment can be a persistent state for newly-under-employed individuals.

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[42]Specifically, we substitute the no-arbitrage condition with the optimal allocation condition (4).

[43]Since there can only be one high-skill applicant in this bargaining configuration, this is true as long as there are not already many high-skilled searching in the low tech-island (i.e., x_h is not too large), which is the case for our parameter values.

To explain these facts, we propose a search model with an endogenous "ranking" mechanism, in which high-skill applicants are systematically hired over less-skilled competing applicants. Some high-skill workers become under-employed in order to escape the competition for high-skill jobs and find a job more rapidly. A quantitative version of the model with aggregate productivity shocks can capture the key features of the under-employment and notably its counter-cyclicality.

Our endogenous ranking mechanism suggests two additional possible benefits of a higher education level. First, if firms prefer more educated workers to less educated ones, more educated workers can extract on average a larger share of a match surplus than less educated workers, i.e., they receive a higher labor income share. Second, a higher education level may not only guarantee a higher expected income but may also provide a lower volatility of income, because highly educated workers can partially smooth out adverse labor demand shocks by moving down the occupational ladder.

Finally, note that the counter-cyclicality of under-employment may offer an explanation for Davis and Wachter (2011)'s finding that the cumulative earnings losses associated with job displacement are (i) substantial and (ii) larger if the displacement occurs during a recession. While under-employment is an optimal choice for high-skill workers in our model, it is easy to imagine cases for which high-skill workers are forced down the occupation ladder, because of borrowing constraints or reputation considerations associated with long unemployment spells (Kroft, Lange and Notowidigdo, 2013). Then, if under-employment is a persistent state and moving back up the ladder is difficult, a high-skill worker who moves down the job ladder following displacement will suffer a persistent drop in income. Exploring this conjecture would be an interesting avenue for future research.

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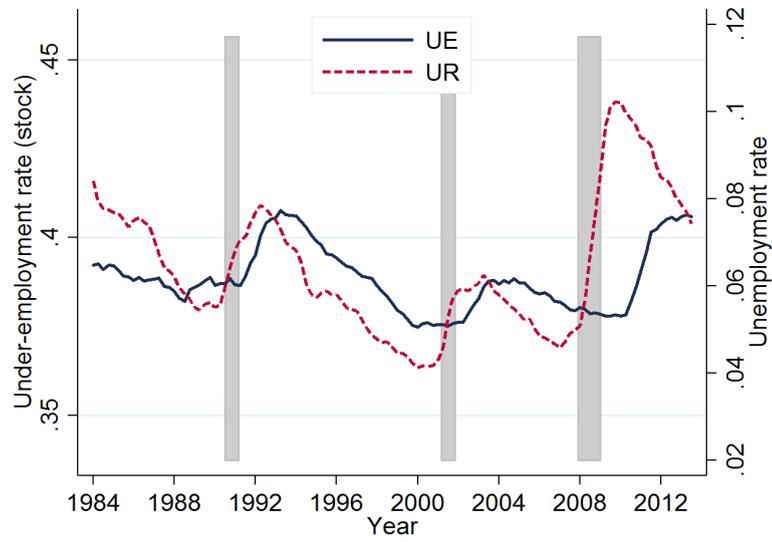
Figure 1. Under-employment in the US

Source: Current Population Survey, 1983–2013. Unemployment rate (dashed line) and Under-employment rate (plain line), defined as the fraction of individuals with some college education (or more) working in occupations requiring at most a high school degree (as defined in the 2012 Occupational Outlook Handbook). The under-employment rate series is cleared from compositional effects, as described in the main text.

(a) Partial equilibrium

(b) General equilibrium
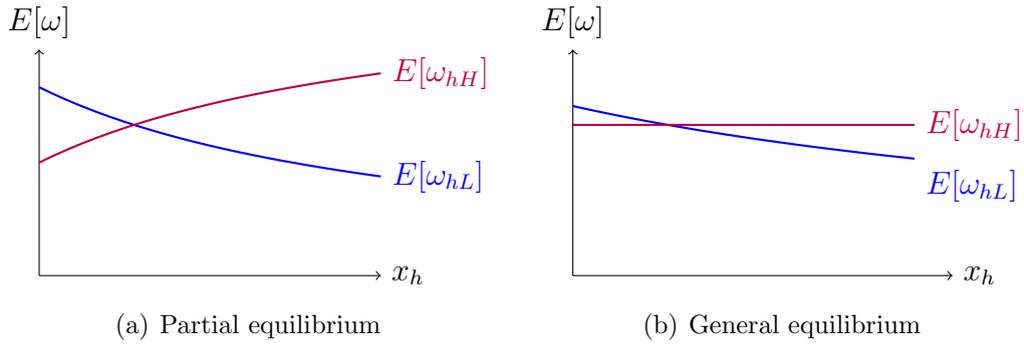
Figure 2. Expected wage of type-h workers in the high-tech island (island H) and in the low-tech island (island L).

(a) Shock $\Delta q_H > 0$
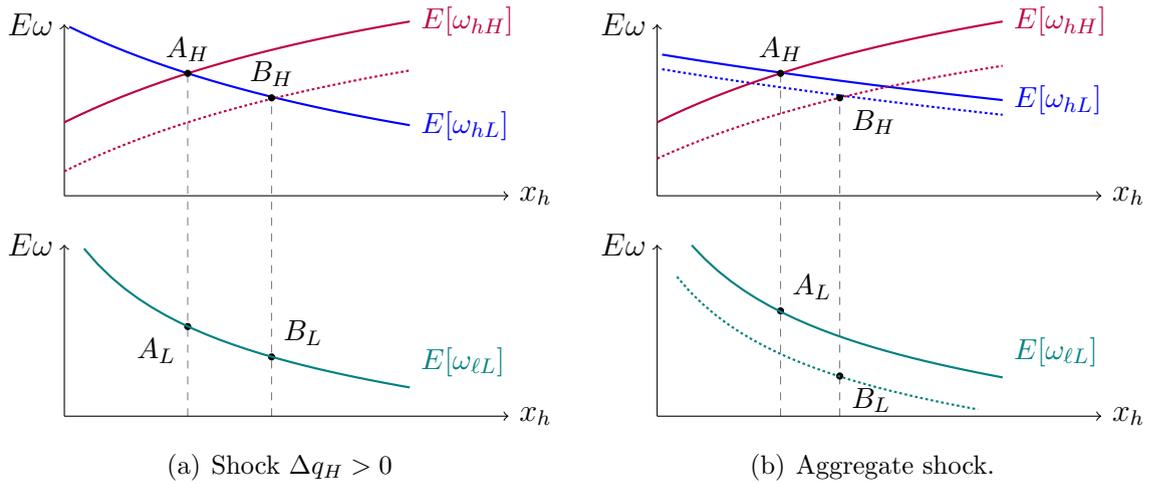
(b) Aggregate shock.

Figure 3. Partial Equilibrium—effects of (a) a shock in the high-tech island (island H), and (b) an aggregate shock.
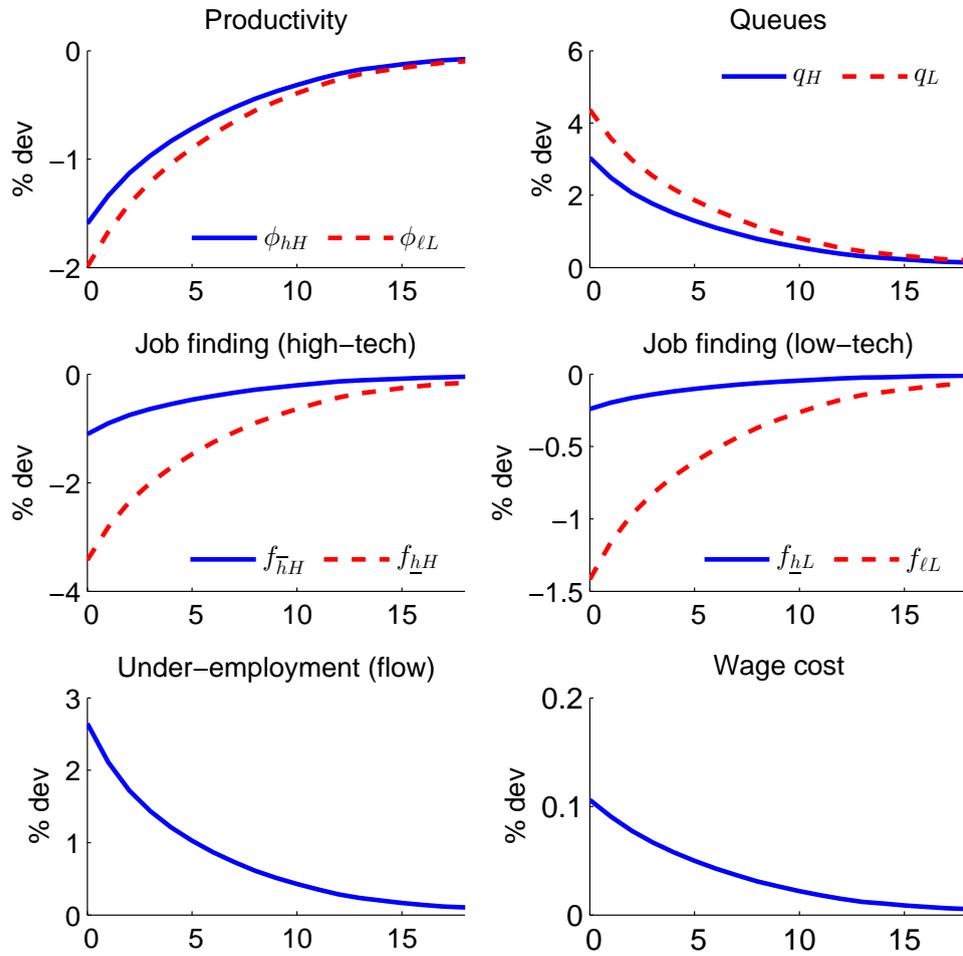
Figure 4. Impulse response functions to an aggregate productivity shock

Notes: Model-based impulse response functions to an aggregate productivity shock for (i) island-specific productivities ($\varphi_{\ell L}$ and φ_{hH}, top left), (ii) "initial" queues (q_L and q_H, top right), (iii) job finding rates in island H for type-$\overline{h}$ and type-$\underline{h}$ ($f_{\overline{h}H}$ and $f_{\underline{h}H}$, middle left), (iv) job finding rates in island L for type-$\underline{h}$ and type-ℓ ($f_{\underline{h}L}$ and $f_{\ell L}$, middle right), (v) fraction of high-skill workers searching in the low-tech island (x_h, bottom left), (vi) wage cost of under-employment ($w_{\underline{h}H}/w_{\underline{h}L}$, bottom-right).

Figure 5. Cross-correlogram between the unemployment rate and the under-employment rate, data (1983–2013) and model.

Notes: Empirical (blue) and model-based (red) cross-correlations between the under-employment and unemployment rates. Confidence intervals for the empirical cross-correlations are calculated by using Fisher's z transform (at the 95% confidence level).
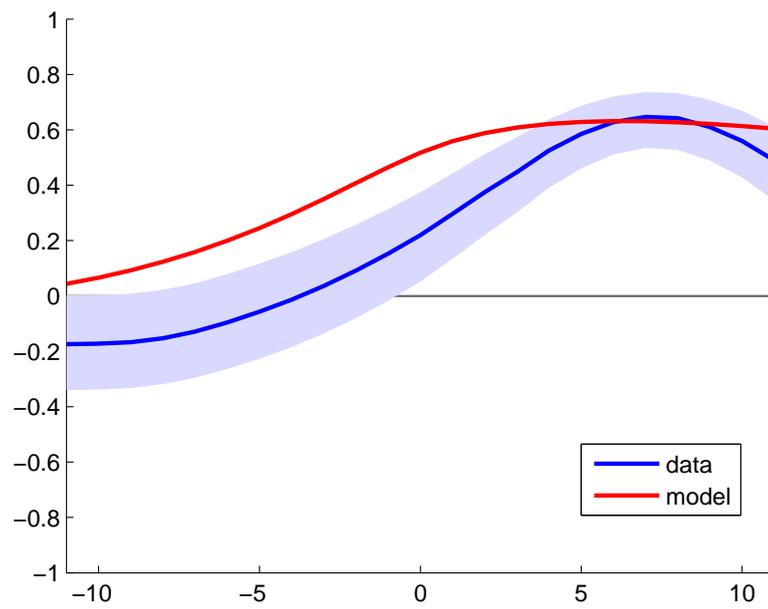
Table 1. Cross-correlations between the unemployment rate and the under-employment rate, 1983–2013.

Lags(-)/Leads(+) of the unemployment rate (quarters)					
	−4	0	+4	+8	+16
Correlation	-.01	.22	.53	.64	-.03
	[-.01,.16]	[.05,.37]	[.39,.64]	[.53,.73]	[-.21,.15]

Source: Current Population Survey, 1983–2013. This table reports the cross-correlations between the detrended unemployment rate and the detrended under-employment rate, defined as the fraction of individuals with some college education (or more) working in occupations requiring at most a high school degree (as defined in the 2012 Occupational Outlook Handbook). The under-employment rate series is cleared from compositional effects, as in Figure 1. Both series are detrended with an HP-filter $\lambda = 10^5$. Confidence intervals in brackets are calculated by using Fisher's z transform (at the 95% confidence level).

Table 2. Permanence of under-employment, 1979–2013.

Employment status one year after hiring	E_L	E_H	U
Fraction	.645	.268	.090

Distribution of newly under-employed workers one year after having been hired in an under-employed occupation. E_L denotes employment in an occupation requiring at most a high school degree (i.e., under-employment), E_H denotes employment in an occupation requiring more than a high school degree, and U denotes unemployment. Sample contains 1,210 observations.

Table 3. Wage differences by occupation degree requirements and education.

Panel A:				
Hiring wage	(1)	(2)	(3)	(4)
Under-employed dummy	-.399	-.319	-.281	-.281
	(.042)	(.047)	(.044)	(.045)
State Fixed Effects	Yes	Yes	Yes	Yes
Date Fixed Effects	Yes	Yes	Yes	Yes
Individual characteristics	No	Yes	Yes	Yes
Past-occupation Fixed Effects	No	Yes	Yes	Yes
Past Wage	No	No	Yes	Yes
Unemployment duration	No	No	No	Yes
Observations	1,321	1,321	1,321	1,321
Panel B:				
Hiring wage	(1)	(2)	(3)	(4)
College-education dummy	.282	.248	.251	.236
	(.013)	(.013)	(.013)	(.018)
Sample	All	All	All	Homog. occ.
State Fixed Effects	Yes	Yes	Yes	Yes
Date Fixed Effects	Yes	Yes	Yes	Yes
Current-occupation Fixed Effects	Yes	Yes	Yes	Yes
Individual characteristics	No	Yes	Yes	Yes
Unemployment duration	No	No	Yes	Yes
Observations	26,302	26,302	26,302	11,882

Source: CPS (1979–2013). In Panel A, the dependent variable is the hiring wage of an unemployed individual with some college education, and previously employed in an occupation requiring more than a high-school degree (as defined in the 2012 Occupational Outlook Handbook). The under-employment dummy equals 1 if the individual has just been hired in an occupation requiring a high school degree or less. In Panel B, the dependent variable is the hiring wage of an unemployed individual (either newly-under-employed or with low education) in an occupation requiring a high school degree or less. The college-education dummy equals 1 if the individual has some college experience. Controls for individual characteristics include a set of dummies for age, sex and marital status. All regressions include a set of dummies for the state of residence, and a set of dummies for the date (quarter/year) of interview. The sample *Homog. occ.* excludes all Census occupations that are further disaggregated in the 2010 Standard Occupational Classification. Robust standard errors are reported in parentheses.

Table 4. Wage differences by education and occupation degree requirements—cyclicality.

Hiring wage of college-educated workers	(1)	(2)
Under-employed dummy	-.358	-.354
	(.013)	(.014)
Under-employed $\times$ Slack$_1$	-.040	
	(.031)	
Under-employed $\times$ Slack$_2$		-.052
		(.029)
State Fixed Effects	Yes	Yes
Date Fixed Effects	Yes	Yes
Individual characteristics	Yes	Yes
Past-occupation Fixed Effects	Yes	Yes
Observations	10,566	10,566

Source: CPS (1979–2013). The dependent variable is the hiring wage of an unemployed individual with some college. The under-employed dummy equals 1 if the individual is hired in an occupation requiring a high school degree or less. Controls for individual characteristics include a set of dummies for age, sex and marital status. All regressions also include a set of dummies for the state of residence, the date (quarter/year) of interview and past-occupation (3-digit). Robust standard errors are reported in parentheses. $Slack_1$ is a dummy equal to 1 if the HP-filtered ($\lambda = 10^5$) unemployment rate is one standard-deviation above its average level. $Slack_2$ is a dummy equal to 1 if (log) vacancy-unemployment is one standard-deviation below its average level. Vacancy posting is measured as the composite Help Wanted index from Barnichon (2010).

Table 5. Parameter values for the benchmark economy.

Panel A: Calibrated parameters		Value	Source
δ	Discount factor	0.996	Interest rate 5% p.a.
s	Separation rate	0.02	CPS, 1979–2013
n_h	Ratio of college-educated workers	0.40	CPS, 1983–2013
β	Bargaining Power	0.50	
b	Replacement ratio	0.20	
$\varphi_{\underline{h}H}$	Productivity (type $\underline{h}$, island H)	1.00	Normalization
τ_L	Cyclical elasticity, island L	1.00	Normalization
τ_H	Cyclical elasticity, island H	0.80	HWOL, 2005–2013
Panel B: Estimated parameters		Value	
$\varphi_{\ell L}$	Productivity (type ℓ, island L)	0.64	
$\varphi_{\underline{h}L}$	Productivity (type $\underline{h}$, island L)	0.80	
$\varphi_{\overline{h}H}$	Productivity (type $\overline{h}$, island H)	1.37	
η_h	Ratio of type $\overline{h}$ to type $\underline{h}$	0.48	
c_L	Vacancy-posting cost, island L	1.48	
c_H	Vacancy-posting cost, island H	10.3	
ν_L	Matching efficiency, island L	0.35	
ν_H	Matching efficiency, island H	0.46	

Table 6. Steady-state moments.

Panel A: Targets					
	Under-empl.	Job finding rates		Wages	
Variable	UE	f_h	f_ℓ	w_{hH}/w_{hL}	$w_{hH}/w_{\ell L}$
Data (1983–2013)	0.38	0.27	0.25	1.28	1.60
Model	0.39	0.27	0.25	1.28	1.59

Panel B: Unobserved moments					
	Job finding rates			Wages	
Variable	$f_{\overline{h}H}$	$f_{\underline{h}H}$	$f_{\underline{h}L}$	$w_{\underline{h}H}/w_{\underline{h}L}$	$w_{\overline{h}H}/w_{\underline{h}H}$
Model	0.31	0.14	0.33	1.10	1.32

Notes: The first row reports the average unemployment and job finding rates over the period 1983–2013, and the wage ratios correspond to the results reported in columns (4)-panel A and (3)-panel B of Table 3. f_h and f_ℓ denote the observed job finding rates of type-h and type-ℓ workers, and $f_{\overline{h}H}$, $f_{\underline{h}H}$ and $f_{\underline{h}L}$ denote the unobserved job finding rates of respectively type-$\overline{h}$ in island H, type-$\underline{h}$ in island H and type-$\underline{h}$ in island L. w_{hH}, w_{hL} and $w_{\ell L}$ denote the observed wages of respectively type-h workers in island H, type-h workers in island L and type-ℓ workers in island L. $w_{\underline{h}H}$, $w_{\underline{h}L}$ and $w_{\overline{h}H}$ denote the unobserved wages of respectively type-$\underline{h}$ in island H, type-$\underline{h}$ in island L and type-$\overline{h}$ in island H.

Table 7. Simulated Moments.

Panel A: Data

Variable x	Under-employment UE	Job finding rates f_ℓ	f_h	Wages w_{hH}/w_{hL}	$w_{hH}/w_{\ell L}$
$\sigma(x)/\sigma(UR)$	0.80	3.08	2.73	2.59	3.69
$\rho_{x,UR}$	0.57†	-0.90	-0.84	0.16	0.38
$\rho_{x,x_{-1}}$	0.90	0.96	0.96	0.78	0.94

Panel B: Model

Variable x	Under-employment UE	Job finding rates f_ℓ	f_h	Wages w_{hH}/w_{hL}	$w_{hH}/w_{\ell L}$
$\sigma(x)/\sigma(UR)$	0.93	3.55	2.63	1.88	2.55
$\rho_{x,UR}$	0.63†	-0.81	-0.81	0.81	0.81
$\rho_{x,x_{-1}}$	0.90	0.96	0.96	0.96	0.96

Notes: The empirical moments are estimated over the period 1983–2013 at a quarterly frequency, and wage series are cleaned from compositional effects (state, age, sex and marital status). Model estimates are obtained from 1,000 simulations of the model over 384 months (or 128 quarters). f_h and f_ℓ denote the job finding rates of type-h and type-ℓ workers, and w_{hH}, w_{hL} and $w_{\ell L}$ denote the (average) wages of respectively type-h workers in island H, type-h workers in island L and type-ℓ workers in island L. $\dagger$: We report the correlation between under-employment and the 4-quarter lag of unemployment. The contemporaneous correlation between unemployment and under-employment is 0.22 in the data and 0.57 in the model. Figure 5 shows the complete cross-correlagram between under-employment and unemployment (empirical and model-based).

Table 8. Counterfactual experiments.

	$E(UE)$	$\frac{\sigma(UE)}{\sigma(UR)}$	$\rho_{UE,UR}$	$\frac{\sigma(f_\ell)}{\sigma(f_h)}$	$\frac{\sigma(w_\ell)}{\sigma(w_h)}$
Baseline	0.39	0.93	0.57	1.35	0.69
Fixed under-employment	0.39	0.00	0.00	1.28	0.66
Efficient under-employment	0.37	0.52	-0.38	1.29	0.67

Notes: Model estimates under the three different scenarios are obtained from 1,000 simulations of the model over 384 months (or 128 quarters). f_h and f_ℓ denote the job finding rates of type-h and type-ℓ workers, and w_{hH}, w_{hL} and $w_{\ell L}$ denote the wages of respectively type-h workers in island H, type-h workers in island L and type-ℓ workers in island L. $E(.)$ and $\sigma(.)$ denote respectively the mean and standard-deviation of a variable.

A   Appendix Figures and Tables

(a) With sectoral fluctuations.
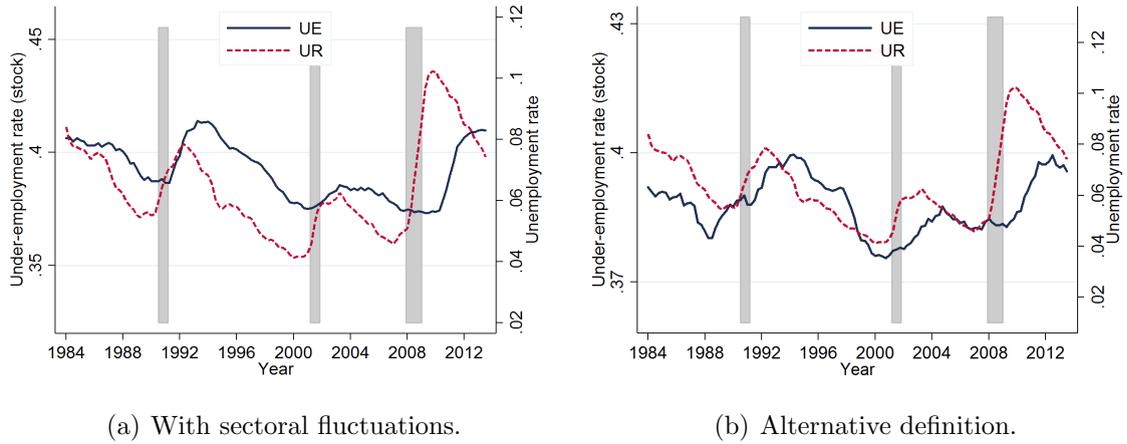
(b) Alternative definition.

Figure A1. Under-employment in the US – sensitivity analysis

Source: Current Population Survey, 1983–2013. In the left panel, the under-employment rate (plain line) is defined as the fraction of individuals with some college education (or more) working in occupations requiring at most a high school degree (as defined in the 2012 Occupational Outlook Handbook). The under-employment rate series is cleared from composition effects (age, gender, marital status, state of residence) and seasonal variations, but not from industry composition effects (unlike our baseline estimate in Figure 1). In the right panel, we define the under-employment rate as the fraction of workers that are over-qualified relative to the typical path of entry into an occupation, measured as the average education level of that occupation during the 1984–1990 sample period.

B Appendix: Under-employment in standard search models

In this section, we argue that it is difficult (given the behavior of the US labor market) to rationalize the existence and the counter-cyclicality of under-employment using search models with random matching (Pissarides, 1985). In particular, explaining the counter-cyclicality of under-employment in a search model with random matching requires that either the wage cost of under-employment is lower in recessions or that finding a job in a low-requirement occupation is easier in recessions. However, none of these two conditions appear to hold in the data.

To see that, consider a labor market divided in two islands; a high-tech island (island H) and a low-tech island (island L). In each island, and this is a key assumption in random matching search models of the labor market, the matching probability is independent of the worker type.

Denote by f_L the job finding rate of a high-skill worker in island L, by U_L (resp. W_{L1}) the value of being unemployed (resp. employed) in island L, and by f_H and U_H (W_H) the corresponding values in the high-tech island H.

As in the search and matching literature with multiple islands (Albrecht and Vroman, 2002) or in the competitive search literature (Moen, 1997), high-skill workers choose in which labor market to look for a job. If under-employment exists in equilibrium, high skill workers must be indifferent between all islands, which implies $U_L = U_H$ or:[44]

$$\frac{f_L}{r + \lambda_L + f_L} \left(w_L - b\right) = \frac{f_H}{r + \lambda_H + f_H} \left(w_H - b\right) \tag{7}$$

in which w_i is the wage, λ_i denotes the job separation rate in island i, b denotes the value of home production including the unemployment benefits, and r denotes the rate at which the future is discounted.

The existence of under-employment   Since under-employment entails a non-negligible wage loss for high-skill workers (Table 3), we have that $w_L < w_H$. As a result, for under-employment to exist in equilibrium (i.e., for equation (7) to be satisfied), high-skill workers must have a higher job finding rate in island 1 than in island 2 ($f_L > f_H$).[45] When matching is random, the job finding rate is identical

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[44]Considering the allocation in steady-state and assuming for ease of exposition that unemployment benefits are identical across islands and that separation rates are identical across islands. Recall that in an island with random search and constant job separation rate λ_i, the value of working in island i is given by $rW_i = w_i + \lambda_i \left(U_i - W_i\right)$ and $rU_i = b + f_i \left(W_i - U_i\right)$, which gives $rU_i = \frac{f_i}{r + \lambda_i + f_i} \left(w_i - b\right)$.

[45]The separation rates are too small to matter ($\lambda_i \ll f_i$, Shimer, 2012), and as shown in Section 1, of the same order of magnitude across islands.

across worker types, so that $f_L > f_H$ implies that it is easier to find a job in a low-requirement occupation *regardless* of one's type.

However, this pattern is at odds with the data. As illustrated in Figure A2, finding a job in a low-requirement occupation takes, on average, just as long as finding a job in a high-requirement occupation.

The counter-cyclicality of under-employment   The higher level of under-employment in recessions (i.e., the counter-cyclicality of under-employment) is difficult to rationalize with standard search models, because the cost of becoming under-employed increases in recessions. Indeed, the wage cost of under-employment is higher in recessions (Table 4), while finding a job in a low-requirement occupation is *not* relatively easier in recessions (Figure A2). In fact, the opposite seems to happen: after controlling for composition (Table 7), the job finding rate in low-requirement occupations is more cyclical than the job finding rate in high-requirement occupations ($\sigma(f_H) > \sigma(f_L)$), making the cost of under-employment even larger in recessions.[46]
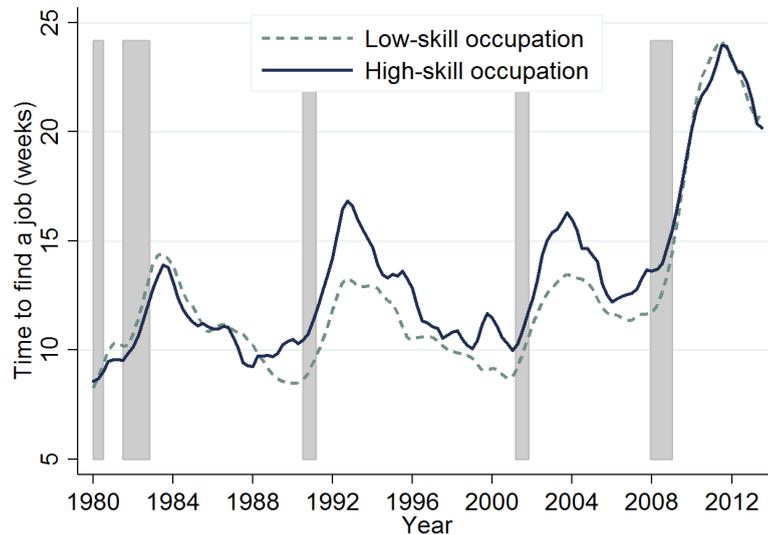

Figure A2. Realized time to find a job (high- versus low-degree requirement occupations).

Source: Current Population Survey, 1979–2013. The realized time to find a job is computed as the average unemployment duration (in weeks) of newly hired workers. The plain line (resp. dashed line) is the realized time to find a job requiring at most a high-school degree (resp. requiring more than a high-school degree).

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[46]Note that if workers were allowed to search simultaneously in both islands (devoting a fraction of their search time to searching in the low-tech island), the random matching model could rationalize the existence of under-employment. However, with $\sigma(f_H) > \sigma(f_L)$ that model would have a hard time rationalizing the counter-cyclicality of under-employment.

C Appendix: Ranking in the data

In this section, we provide some empirical evidence supporting the notion of ranking and the idea that firms prefer the most qualified workers when facing multiple applicants.[47]

Using experimental data compiled by Kroft, Lange and Notowidigdo (2013), we show that when workers apply for a low-qualification job, workers with stronger resumes are substantially more likely to be called back by the firm than workers with just the required qualifications. Moreover, the rank of the candidate in the queue of applications influences the call-back rate beyond the sheer quality of the resume. This finding is consistent with the idea of ranking, but not with the idea of screening, in which only the quality of the resume should matter.[48]

Kroft, Lange and Notowidigdo (2013) examine the effect of unemployment duration on callback rates using fictitious job applications in which a set of quality indicators are manipulated exogenously. Fictitious resumes were varied in their quality, and four resumes (two of a high type, and two of a low type) were sent to the same job posting. A "low-quality" resume has the minimum qualifications required for the job (in terms of experience and education), while a "high-quality" resume has qualifications that exceeded these minimum, with extra years of experience and an extra level of education.[49]

Using this experimental setup, it is possible to test two important predictions of ranking.

First, over-qualified resumes should face higher call back rates than "just-qualified" resumes competing for the same position. To test this prediction, we estimate three different specifications based on the following equation:

$$C_{i,k} = \alpha + \beta D_i + \delta \mathbf{X}_{i,k} + \varepsilon_{i,k}, \qquad (8)$$

where i denotes an applicant and k denotes a job posting. $C_{i,k}$, our dependent variable, is a callback dummy and D_i, our main explaining variable, is a dummy for high-quality resume. The vector of controls $\mathbf{X}_{i,k}$ includes individual i characteristics

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[47]The notion of ranking is also consistent with qualitative evidence on firms' recruitment behavior, with firms typically interviewing many applicants and hiring the most qualified candidate (Barron, Bishop and Dunkelberg, 1985; Barron and Bishop, 1985; Raza and Carpenter, 1987; Behrenz, 2001).

[48]With screening, hiring is determined by a threshold level above which a candidate is hired, and over-qualified candidates are more likely to be above that threshold, irrespective of other candidates. By contrast, with ranking, firms select their new hire among the pool of applicants. Thus, the hiring rate depends on both the number and the quality of the other candidates.

[49]For instance, if the job requires high school completion, the resume would list an associate's degree.

(age and gender) and some job opening k characteristics (type of job and city or job posting fixed effects depending on the specifications).

In the first specification, we only control for the individual characteristics and the three main categories of job (administrative, sales, customer services). In the second specification, we also control for the local labor market by including city fixed effects. Finally, in our third (and preferred) specification, we also include a job posting fixed-effect, which allows us to compare resumes in competition for the same job opening. Table A1 presents the results and shows that, in all 3 specifications, over-qualified resumes have a callback rate that is about 35 percent higher than just-qualified resumes. While the mapping from callback rates to job finding propensities is not straightforward, this evidence strongly suggests that firms do rank candidates by resume's quality, so that higher-educated job seekers likely have an easier time finding a job in a lower requirement occupation than their lower-educated peers.

Second, the rank of an applicant A should matter beyond the sole quality of his resume. In other words, the quality of the other applicants should influence the call-back rate of A above and beyond the quality of A's resume. To exploit variations in the quality of the other applicants, we use a factor that is randomly-drawn across individuals: unemployment duration (see Kroft, Lange and Notowidigdo, 2013). We then test whether the rank (in terms of unemployment duration) of a candidate among the four applicants affect the call-back rate above-and-beyond the resume's quality.[50] The last column of Table A1 shows that this is indeed the case, consistent with the implications of ranking. Adding an individual's rank to our baseline specification, we can see that there is an additional, non-negligible, call-back premium for being ranked higher than the other applicants. This result is compatible with the prediction of a model with ranking but not with a model with screening.

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[50]We also control for the individual unemployment duration and the average unemployment duration of the three other applicants, as this could also affect the average call-back rate.

Table A1. Callback rate as a function of applicants' type.

Call-back rate	(1)	(2)	(3)	(4)
High-type dummy	.0183***	.0172***	.0176***	.0149**
	(.0056)	(.0055)	(.0060)	(.0062)
Rank				-.0074**
				(.0032)
Individual controls	Yes	Yes	Yes	Yes
Fixed-effects		City	Ad	City
Unemployment duration				Yes
Observations	11,648	11,648	11,648	11,648

Significantly different than zero at * 90% confidence, ** 95% confidence, *** 99% confidence. The dependent variable is a dummy equal to one if the applicant received a call-back. The high-type dummy equals 1 if the resume is of high-quality. Rank denotes how the candidate ranks (in terms of unemployment duration) among the four candidates. The controls are gender, age, individual unemployment duration, average unemployment duration across the four applicants and a set of dummies capturing job type. Standard errors between parentheses are clustered at the ad level. The average callback rates in the sample is .044.